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Thermal Action and Specific Heat of the Five-Dimensional Non-Extremal Black Hole

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Abstract

We construct the Euclidean on-shell action for the five-dimensional non-extremal black hole with multiple electric charges. We show that this thermal action agrees with one half of the entropy. This agreement is argued to be related to the generalized Smarr formula of the five-dimensional black hole mass. Through the calculation of the specific heat far off extremality we observe that a phase transition occurs.

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Recently there has been considerable attention to the study of the black hole thermodynamics in string theory. The microscopic degeneracies of black holes were explained from the D-brane bound states. The Bekenstein-Hawking entropy of a certain extremal five-dimensional Reissner-Nordsröm black hole was computed by counting the asymptotic degeneracy of BPS oscillations of an effective string that describes the bound state of D-5-branes and D-1-branes [1]. The computations of the entropies were further performed for the near-extremal six-dimensional black string [2] and the near-extremal five-dimensional black holes [3, 4] with multiple charges, which were shown to be associated with the degeneracy of low-lying non-BPS oscillations of the effective string. The remarkable statistical derivations of the entropies were extended to the four-dimensional black holes [5, 6] and the rotating black holes [7, 8].

On the other hand since Gibbons and Hawking [9] showed that through the saddle-point approximation the Euclidean partition function for quantum gravity is interpreted as the thermal partition function for a system of the black hole temperature, the thermodynamic properties of various black holes were investigated. Based on such direct calculation of the partition function the thermodynamics was constructed for the four-dimensional rotating black holes [10] or the four-dimensional $\mathrm{U}(1)^2$ dilaton black holes [11], where the rule that the entropy is one quarter of the area of the event horizon was discussed. Recently to study the zero mass black holes the four-dimensional anti-Maxwell dilaton Einstein theory as well as the usual Maxwell one, that are specified by a dimensionless dilaton coupling parameter a, were considered [12] and the on-shell actions for the electrically singly-charged non-extremal black holes were evaluated to be equal to the entropies. The specific heats characterized by a were shown to behave differently between both theories.

We will investigate the five-dimensional non-extremal black hole with multiple electric charges by means of the Gibbons-Hawking method. We will derive the Euclidean on-shell action and discuss the thermal properties of this five-dimensional black hole, which will be compared with those of the four-dimensional charged dilaton black holes. The specific heat will be computed for the non-extremal black hole as well as the near-extremal one, from which the occurrence of a phase transition will be suggested.

We start with the six-dimensional low-energy Lorentzian action for the type IIB string theory in the canonical frame

$$\frac{1}{16\pi G_6}\int d^6x\sqrt{-g}\left(R-(\nabla\phi)^2-\frac{1}{12}e^{2\phi}H^2\right), \tag{1}$$

where H is the RR three-form field strength [2]. Through the black string solution specified by

$$ds_6^2 = e^{2D}(dx_5 + A_\mu dx^\mu)^2 + ds_5^2, \tag{2}$$

where x^μ are the coordinates of the five-dimensional spacetime and x_5 is on an S^1 of

circumference L, the action is truncated into the five-dimensional one

$$\frac{1}{16\pi G_5}\int d^5x\sqrt{-g}e^D\left(R-(\nabla\phi)^2-\frac{2}{3}(\nabla D)^2-\frac{e^{-2D+2\phi}}{4}H_+^2-\frac{e^{-2D-2\phi}}{4}H_-^2-\frac{e^{2D}}{4}G^2\right) \tag{3}$$

with $G_5 = G_6/L$. We set the six-dimensional Newton constant $G_6 = 1$. This action has three U(1) gauge fields such as the usual Kaluza-Klein field strength $G = dA, (H_+)_{\mu\nu} = H_{\mu\nu 5}$ and $H_- = e^{2\phi+D} * H$ where $*$ is the five-dimensional Hodge dual. Here we write down the non-extremal six-dimensional black string solution in the canonical frame

$$\begin{aligned} ds_6^2 = &-\left[1-\left(\frac{r_+^2\cosh^2\alpha-r_-^2\sinh^2\alpha}{r^2}\right)\right]dt^2+\sinh 2\alpha\frac{r_+^2-r_-^2}{r^2}dtdx_5 \\ &+\left[1-\left(\frac{r_-^2\cosh^2\alpha-r_+^2\sinh^2\alpha}{r^2}\right)\right]dx_5^2+\left(1-\frac{r_-^2}{r^2}\right)^{-1}\left(1-\frac{r_+^2}{r^2}\right)^{-1}dr^2+r^2d\Omega_3^2 \end{aligned} \tag{4}$$

with a boost parameter α, which is constructed by boosting the non-extremal zero-momentum black string solution found in [13], where the event horizon and the inner horizon are located at $r = r_+, r_-$ respectively. Comparison of this expression with (2) leads to

$$e^{2D} = 1-\frac{r_-^2\cosh^2\alpha-r_+^2\sinh^2\alpha}{r^2},\ A_t = e^{-2D}\sinh 2\alpha\frac{r_+^2-r_-^2}{2r^2}. \tag{5}$$

Performing the conformal rescaling as $g_{\mu\nu} = e^{-2D/3}g^c_{\mu\nu}$ for the five-dimensional action (3) we get

$$\frac{L}{16\pi}\int d^5x\sqrt{-g_c}\left(R-(\nabla\phi)^2-2(\nabla D)^2-\frac{e^{-4D/3+2\phi}}{4}H_+^2-\frac{e^{-4D/3-2\phi}}{4}H_-^2-\frac{e^{8D/3}}{4}G^2\right). \tag{6}$$

The five-dimensional black hole with electric charges about both H_+ and H_-

$$Q_+ \equiv \frac{1}{8}\int_{S^3}e^{-4D/3+2\phi} * H_+,\ Q_- \equiv \frac{1}{4\pi^2}\int_{S^3}e^{-4D/3-2\phi} * H_- \tag{7}$$

can also carry the U(1) charge associated with G

$$P \equiv \frac{2\pi n}{L} = \frac{L}{16\pi}\int_{S^3}e^{8D/3} * G, \tag{8}$$

which is also regarded as the ADM momentum around S^1 for the six-dimensional black string. The parameters $r_\pm$ are related to the charge by $Q^2 \equiv 2Q_+Q_- = (\pi r_+ r_-)^2$. This non-extremal black hole solution is characterized by

$$\begin{aligned} \phi &= \phi_h, && (9) \\ e^{-4D/3+2\phi} * H_+ &= \frac{4Q_+}{\pi^2}\epsilon_3,\ e^{-4D/3-2\phi} * H_- = 2Q_-\epsilon_3, && (10) \\ ds_{5c}^2 &= -f^{-2/3}\left(1-\frac{r_0^2}{\hat{r}^2}\right)dt^2+f^{1/3}\left[\left(1-\frac{r_0^2}{\hat{r}^2}\right)^{-1}d\hat{r}^2+\hat{r}^2d\Omega_3^2\right], && (11) \end{aligned}$$

where $r_0^2 = r_+^2 - r_-^2$ and ϵ_3 is the volume form on the unit three-sphere S^3. Using the radial coordinate $\hat{r}^2 = r^2 - r_-^2$, we express e^{2D} as

$$e^{2D} = \frac{1 + r_0^2 \sinh^2 \alpha / \hat{r}^2}{1 + r_-^2/\hat{r}^2} \tag{12}$$

and derive the five-dimensional metric (11) with $f = (1 + r_-^2/\hat{r}^2)^2(1 + r_0^2 \sinh^2 \alpha/\hat{r}^2)$ from the six-dimensional metric by the dimensional reduction along x_5 direction through (2), (4) and $ds_5^2 = e^{-2D/3} ds_{5c}^2$. The equation of motion for ϕ

$$\nabla^2 \phi - \frac{1}{4} e^{-4D/3+2\phi} H_+^2 + \frac{1}{4} e^{-4D/3-2\phi} H_-^2 = 0 \tag{13}$$

is satisfied by a special asymptotic value ϕ_h specified by

$$e^{2\phi_h} = \frac{2Q_+}{\pi^2 Q_-}, \tag{14}$$

since H_+, H_- are simultaneously obtained from (10) as

$$H_{t\hat{r}}^+ = \frac{2Q_-}{\hat{r}^3(1 + r_-^2/\hat{r}^2)^2}, \; H_{t\hat{r}}^- = \frac{4Q_+/\pi^2}{\hat{r}^3(1 + r_-^2/\hat{r}^2)^2}. \tag{15}$$

From the expression of A_t in (5) the U(1) field strength is determined in a similar form as

$$G_{t\hat{r}} = \frac{r_0^2 \sinh 2\alpha}{\hat{r}^3(1 + r_0^2 \sinh^2 \alpha/\hat{r}^2)^2}, \tag{16}$$

which combines with (8) to yield

$$P = \frac{\pi L}{8}(r_+^2 - r_-^2) \sinh 2\alpha. \tag{17}$$

The area A of the event horizon which is located at $\hat{r} = r_0$ corresponding to $r = r_+$ can be calculated from the metric (11) and the entropy of the five-dimensional black hole is given by

$$S = \frac{A}{4G_5} = \frac{L}{2}\pi^2 r_+^2 \sqrt{r_+^2 - r_-^2} \cosh \alpha. \tag{18}$$

This agrees with the entropy of the six-dimensional black string computed from the metric (4).

Now in order to take the semiclassical approximation of the path integral in the Euclid spacetime the above five-dimensional black hole solution must be analytically continued to Euclidean time. Since the Einstein-Hilbert action includes a boundary term which is formally infinite, we must perform an appropriate subtraction [9, 14]. Recently the entropy of the Schwarzschild black hole in the presence of a string instanton has been

calculated [15]. Following the procedure in Ref.[15], we will make the subtraction in the computation of the thermal on-shell action. The subtracted total Euclidean action is given by

$$
\begin{aligned}
I &= I_0 + I_b, && (19)\\
I_0 &= -\frac{L}{16\pi}\int d^5x\sqrt{g_c}(R-(\nabla\phi)^2-2(\nabla D)^2-\frac{e^{-4D/3+2\phi}}{4}H_+^2 \\
&\quad -\frac{e^{-4D/3-2\phi}}{4}H_-^2-\frac{e^{8D/3}}{4}G^2), && (20)\\
I_b &= -\frac{L}{8\pi}\int_{\partial M}\sqrt{h}K+\frac{L}{8\pi}\int_{(\partial M)_\infty}\sqrt{h_0}K_0, && (21)
\end{aligned}
$$

where K is the trace of the extrinsic curvature on the boundary ∂M of the five-dimensional manifold M, h the determinant of the induced metric and K_0, h_0 the corresponding ones at infinity in flat spacetime. The conical singularity at $\hat{r} = r_0$ in (11) is removed by requiring the Euclidean time τ to be identified with period β and Bekenstein-Hawking temperature is obtained by

$$T \equiv \frac{1}{\beta} = \frac{\sqrt{r_+^2 - r_-^2}}{2\pi r_+^2 \cosh\alpha}, \tag{22}$$

which is compared with that of the six-dimensional black string $T_6 = \sqrt{r_+^2 - r_-^2}/(2\pi r_+^2)$ that is derived from (4). Contracting the Einstein equation with the metric tensor we have

$$R = (\nabla\phi)^2 + 2(\nabla D)^2 + \frac{1}{12}(e^{-4D/3+2\phi}H_+^2 + e^{-4D/3-2\phi}H_-^2 + e^{8D/3}G^2). \tag{23}$$

Substitution of it into (20) yields

$$I_0 = \frac{L}{16\pi}\int d^5x\sqrt{g_c}\frac{1}{6}\left(e^{-4D/3+2\phi}H_+^2 + e^{-4D/3-2\phi}H_-^2 + e^{8D/3}G^2\right), \tag{24}$$

where the dilaton terms such as $(\nabla\phi)^2, (\nabla D)^2$ have disappeared. In the Euclid spacetime the U(1) field strengths (15), (16) become pure imaginaries. Therefore putting them into (24) and combining with (12), (14) we find the volume integration to be simplified into the symmetric two terms as

$$I_0 = -\frac{L}{16\pi}\frac{\beta\omega_3}{6}\int_{r_0}^{\infty}\frac{d\hat{r}}{\hat{r}^3}\left(\frac{32Q_+Q_-}{\pi^2(1+r_-^2/\hat{r}^2)^2} + \frac{2r_0^4\sinh^2 2\alpha}{(1+r_0^2\sinh^2\alpha/\hat{r}^2)^2}\right), \tag{25}$$

where the two contributions from H_+^2 and H_-^2 have been equal and ω_3 is the three-dimensional unit-sphere volume. This integration can be carried out to be

$$I_0 = -\frac{L\beta\omega_3}{24\pi}(2r_-^2 + r_0^2\sinh^2\alpha). \tag{26}$$

Here to evaluate the boundary term I_b for the non-extremal Euclidean black hole solution whose boundary is at the infinity, we look into the asymptotic behavior of the metric for large $\hat{r}$

$$ds^2 \sim \left(1 - \frac{2R_0^2 + r_0^2}{\hat{r}^2}\right) d\tau^2 + \left(1 + \frac{R_0^2 + r_0^2}{\hat{r}^2}\right) d\hat{r}^2 + (\hat{r}^2 + R_0^2) d\Omega_3^2, \tag{27}$$

where $R_0^2 = (2r_-^2 + r_0^2 \sinh^2 \alpha)/3$. Further the metric shifted by $\hat{r}^2 + R_0^2 = R^2$ is written as

$$ds^2 \sim \left(1 - \frac{2R_0^2 + r_0^2}{R^2}\right) d\tau^2 + \left(1 + \frac{2R_0^2 + r_0^2}{R^2}\right) dR^2 + R^2 d\Omega_3^2. \tag{28}$$

The ADM mass of this solution reads

$$M = \frac{3\omega_3}{16\pi G_5}(2R_0^2 + r_0^2), \tag{29}$$

which is expressed as

$$M = \frac{L\pi}{8}[2(r_+^2 + r_-^2) + \cosh 2\alpha (r_+^2 - r_-^2)]. \tag{30}$$

From a spacelike unit vector $n_\mu = (0, N, 0, 0, 0)$ normal to the boundary surface $R = R_\infty$, where $N = \sqrt{g_{RR}} = (1 + (2R_0^2 + r_0^2)/R^2)^{1/2}$ is the lapse function, we calculate the trace of the extrinsic curvature on the boundary $K = h^{\mu\nu}\nabla_\mu n_\nu = (g^{RR}\partial_R g_{RR} + \sum_i g^{\theta_i\theta_i}\partial_R g_{\theta_i\theta_i})/(2N)$ with the induced metric on the boundary $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$ and obtain

$$\frac{L}{8\pi}\int_{\partial M}\sqrt{h}K = \frac{L\beta\omega_3}{16\pi}\left[6R_\infty^2\left(1 - \frac{2R_0^2 + r_0^2}{R_\infty^2}\right) + 2(2R_0^2 + r_0^2)\right]. \tag{31}$$

The subtracted term is derived for the flat metric

$$ds^2 \sim \left(1 - \frac{2R_0^2 + r_0^2}{R^2}\right) d\tau^2 + dR^2 + R^2 d\Omega_3^2 \tag{32}$$

in a similar fashion to be

$$\frac{L}{8\pi}\int_{(\partial M)_\infty}\sqrt{h_0}K_0 = \frac{L\beta\omega_3}{16\pi}6R_\infty^2\left(1 - \frac{2R_0^2 + r_0^2}{2R_\infty^2}\right). \tag{33}$$

Gathering together we have $I_b = \beta M/3$. Since an interesting relation $ST = L\pi r_0^2/4$ makes I_0 (26) expressed as

$$I_0 = \beta\left(-\frac{M}{3} + \frac{1}{2}ST\right), \tag{34}$$

we arrive at the Euclidean on-shell action

$$I = \frac{1}{2}S. \tag{35}$$

The expression I_0 (26) is alternatively described by

$$I_0 = -\frac{\beta}{3}(Q\Phi_Q + P\Phi_P), \tag{36}$$

where $\Phi_Q = LQ/2r_+^2$ is the electric potential on the horizon in five dimensions [12, 16] and $\Phi_P = 4P/L(r_0 \cosh\alpha)^2$ may be regarded as the electric potential associated with G. To consider the thermodynamics of the five-dimensional black hole we define the Gibbs free energy

$$W = M - TS - Q\Phi_Q - P\Phi_P = -T\log Z. \tag{37}$$

At finite temperature the classical Euclidean action I can be related approximately to a thermal partition function Z as $Z \sim e^{-I}$, which yields

$$W = IT = \frac{1}{2}ST. \tag{38}$$

Combining them we obtain

$$M = \frac{3}{2}ST + Q\Phi_Q + P\Phi_P, \tag{39}$$

which is also seen from (34) and (36). In Ref.[16] the electrically singly-charged d-dimensional black hole solution in the Maxwell dilaton Einstein theory was constructed and the generalized Smarr formula $M = ((d-2)/(d-3))ST + Q\Phi_Q$ was presented. Our one result (39) is considered to exhibit the generalized Smarr formula in five dimensions, which corresponds to the Smarr formula $M = 2ST + Q\Phi_Q$ for the four-dimensional electrically singly-charged dilatonic black hole [12]. The other result that the Euclidean on-shell action is one half of the entropy in five dimensions is compared with the four-dimensional case where it equals to the entropy itself [11, 12]. As seen above the two results are consistently related to each other.

Now let us consider first the near-extremal black hole by parametrizing the left-moving and right-moving oscillations of a fundamental string around S^1 as

$$n_L \equiv \frac{e^{-2\alpha}}{2\sinh 2\alpha} n \sim n', \; n_R \equiv \frac{e^{2\alpha}}{2\sinh 2\alpha} n \sim n + n' \tag{40}$$

to keep the total charge $n = n_R - n_L$ fixed in the large α region with small r_0. The mass and entropy of it are expressed as $M = LQ/2 + (n+2n')/R, S = \sqrt{2}\pi Q(\sqrt{n+n'} + \sqrt{n'})$ with $L = 2\pi R$. This near-extremal entropy has been understood from a counting of the underlying microscopic degree of freedom by using the weak-coupling D-brane description [2, 3, 4]. Using these expressions we can demonstrate a thermal relation $dM = TdS$ with $T = 2\sqrt{2n'(n+n')}/(\pi QR(\sqrt{n+n'} + \sqrt{n'}))$. This Bekenstein-Hawking temperature is further described in terms of the right-moving temperature $T_R = \sqrt{2(n+n')}/\pi QR$ and

the left-moving one $T_L = \sqrt{2n'}/\pi QR$ as a harmonic mean $2/T = 1/T_R + 1/T_L$ [17]. The specific heat at constnat charges Q, P is evaluated as

$$C = \left(\frac{\partial M}{\partial T}\right)_{Q,P} = \frac{S}{(n+2n')/\sqrt{n'(n+n')} - 1}. \tag{41}$$

This tells us that near extremality the specific heat is positive and when we approach the extremality it goes to zero.

We will look more closely at the specific heat far off extremality. In doing so, we work on the non-extremal black hole solution parametrized by r_+, r_- and α. Instead choosing Q, P and α as the independent parameters we can express $r_\pm, M$ as

$$r_\pm^2 = \pm\frac{4P}{\pi L \sinh 2\alpha} + \sqrt{D} \tag{42}$$

with $D = (Q/\pi)^2 + (4P/\,\pi L \sinh 2\alpha)^2$,

$$M = \frac{L\pi}{2}\left(\sqrt{D} + \frac{2P}{\pi L \tanh 2\alpha}\right). \tag{43}$$

Taking the derivative of (18) expressed in terms of Q, P, α as

$$S = \frac{L\pi^2}{2}\sqrt{\frac{4P}{\pi L \tanh\alpha}}\left(\sqrt{D} + \frac{4P}{\pi L \sinh 2\alpha}\right) \tag{44}$$

with respect to α at constant Q, P we can obtain a factorized form

$$\frac{\partial S}{\partial \alpha} = -\frac{L\pi^2}{2}\sqrt{\frac{4P}{\pi L \tanh\alpha}}\frac{(\sqrt{D}\sinh 2\alpha + \frac{4P}{\pi L})(\sqrt{D}\sinh 2\alpha + \frac{8P}{\pi L}\cosh 2\alpha)}{\sqrt{D}\sinh^3 2\alpha}, \tag{45}$$

which produces $dM = TdS$ again with

$$T = \frac{\sqrt{4P\tanh\alpha/\,\pi L}}{\pi(4P/\,\pi L + \sqrt{D}\sinh 2\alpha)}, \tag{46}$$

which is given from (22). The specific heat is also computed from (43), (46) to be

$$C^{-1} = \frac{1}{S}\left(\frac{2\sqrt{D}\cosh 2\alpha}{\sqrt{D} + 8P/(\pi L \tanh 2\alpha)} - 1\right), \tag{47}$$

which is in a similar form to (41). In the large α region the specific heat reduces to the near-extremal positive one (41) through $e^{2\alpha} \sim \sqrt{(n+n')/n'}$ which is given from (40), while in the small α region it turns out to be negative. Therefore the specific heat blows up at the point $\alpha = \alpha_0$ provided by the solution of

$$f(x) \equiv Q^2(x^2-1)(2x-1)^2 - \left(\frac{4P}{L}\right)^2(4x-1) = 0 \tag{48}$$

with $f(1) < 0$ for $x = \cosh 2\alpha \geq 1$. At the particular black hole mass specified by α_0 there happens a phase transition. For the zero charge $Q = 0$ with $P \neq 0$ we see that the specific heat becomes always negative

$$C = -L\pi^2 \left(\frac{4P}{\pi L}\right)^{3/2} \frac{1 + 2\cosh 2\alpha}{\sinh 2\alpha \sqrt{\tanh \alpha}}, \tag{49}$$

whose expression has already appeared in the above small α region with nonzero Q, P. This sign is characteristic of the Kaluza-Klein black holes [18]. On the other hand in the small P region with nonzero Q and finite α the specific heat takes small positive value. Therefore we can say that the existence of both Q and P charges is responsible for the appearance of the singularity. This singularity is compared with that in the four-dimensional $\mathrm{U}(1)^2$ dilaton black hole, where the singularity appears only when it has both electric and magnetic charges [19]. Similarly in the four-dimensional electrically singly-charged black hole paramertrized by a, which characterizes the coupling of the dilaton to the gauge field, the phase transition was found only for $a^2 < 1$ [12, 16]. The specific heats for the $a = 1$ stringy black hole and the $a = \sqrt{3}$ Kaluza-Klein black hole are always negative, which are considered to correspond to the disappearances of singularities for the $\mathrm{U}(1)^2$ dilaton black hole with only electric charge [19] and our $Q = 0, P \neq 0$ case in (49) respectively.

In conclusion we have made the Euclidean semi-classical path-integral analysis of the thermodynamics of the five-dimensional non-extremal black hole with multiple electric charges. We have performed the explicit calculation of the on-shell action with the suitably subtracted boundary term and found that it is one half of the black hole entropy. This semi-classical result has been shown to be consistently connected with the generalized Smarr formula for the five-dimensional black hole mass. We expect such connections to appear in the other dimensions. We have carried out the computation of the specific heat of the non-extremal black hole, which was guided by that of the near-extremal one, and observed that there exists a phase transition. It would be interesting to understand this phase transition presented by the strong-coupling black hole description from the alternative weak-coupling D-brane description.

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